VISCOUS FRACTURE CRITERION IN THE PLASTIC MOLDING OF METALS

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1. Despite the fact that plastic treatment of metals is ordinarily realized under conditions of maximum or sufficiently high plasticity, the phenomenon of viscous fracture is often observed during forging, pressing, rolling, and other technological processes. Different attempts at the construction of viscous fracture criteria are known, some of which are based on postulation of certain heuristic principles of cumulative damage [1, 2], and others on the use of special models of the fracture mechanism [3-5]. This latter approach is most expedient since it permits utilization of the results of fine experimental investigations of the microphenomena during fracture, and at least in principle, thereby allows the possibility of perfecting the physical content of the model. As regards the viscous fracture mechanism of sufficiently plastic metals, it is then defined by successively developing processes of origination, growth, and coalescence of pores at submicro-, micro-, and macrolevels, respectively [6]. The mentioned sequence of events, underlying which are phenomena of different scales, cannot be described on the basis of a single approach, and statistical averaging of the preceding levels is used in going over to ever-increasing levels [7].

Averaging of the phenomena of submicroscopic nature that are associated with the processes of origination, motion, and interaction of dislocations and vacancies in the grain scale can be replaced by an equivalent system of internal microstresses without a detailed analysis of the local nature of their distribution [8]. Pore formation is observed at the microlevel near the boundaries of grains, twins, solid inclusions, and other defects and irregularities of the structure. The pores grow approximately isotropically during further deformation and do not interact noticeably. However, after the achievement of a definite pore size, the microdeformation becomes substantially localized, resulting in coalescence and rapid growth of macrocracks. For viscous fracture under plastic metal treatment, the contribution of the last stage to the general history of deformation is not essential, which permits the examination to be limited to pore evolution at the microlevel, and the fracture criterion to be formulated from the condition of reaching the critical porosity. The usual assumptions about the possibility of neglecting elastic deformations as compared with plastic, and discarding taking account of the crystallographic orientation of individual grains, yield a further simplification. By this means the analysis of viscous fracture can be reduced to an investigation of the limit state of elementary structural cells of a rigidly plastic material with pores. However, attempts at an analytical solution of the problem mentioned do not result in visible results even under quite strong constraints, which force reliance on different approximations at definite stages of the solution [4-6].

A detailed analysis of the state in the neighborhood of individual pores is replaced in this paper by a statistical description of the behavior of their ensemble by using the equations of the phenomenological theory of plasticity of porous bodies [9] with the main singularities of the physical model of viscous fracture taken into account. Such an approach results in a sequential derivation of the viscous fraction criterion in macroscopic form, and in its direct comparison with parameters of the deformation process.

2. We shall henceforth consider the following volume of material: Δw, volume of an elementary structural cell; w, an element of the macrovolume containing a sufficiently large number N of cells within which the distribution of macroscopic stresses and strains can be assumed homogeneous; and V, volume of the body ($\Delta w \ll w \ll V$).

The ensemble of pores N in the volume w is defined by the mean porosity

$$v = \frac{1}{N}\sum_{i=1}^{N}\frac{\Delta w_{ni}}{\Delta w_i}, \tag{2.1}$$

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where Δw_{ni} is the volume of the individual pores, and Δw_i is the volume of the structural cells. The mean porosity (2.1) is taken as the single scalar material damage characteristic at the stage of isolated pore growth. The condition of plasticity of the isotropic porous body [9] can be used for the volume w:

$$f = \gamma T^2 + 9\alpha \langle\sigma\rangle^2 - \sigma_s^2 = 0,\ T = (\langle\sigma_{ij}\rangle\langle\sigma_{ij}\rangle)^{1/2},\ \langle\sigma\rangle = \frac{1}{3}\sigma_{ll}, \tag{2.2}$$

where $\gamma = \gamma(v)$, $\alpha = \alpha(v)$ are functions of the material porosity; T, $\langle\sigma\rangle$, intensity of the tangential stresses and the hydrostatic pressure of the averaged field of effective stresses, $\langle\sigma_{ij}\rangle$ in the neighborhood of the pores; σ_s, yield point of the corresponding compact material ($v = 0$). Analogously to [9] the functions $\gamma(v)$, $\alpha(v)$ are found from the condition of the limit state of an elementary structural cell of the material with an equivalent porosity v under pure shear ($\langle\sigma\rangle = 0$) and hydrostatic loading ($T = 0$); for $\gamma(0) = 3$, $\alpha(0) = 0$, Eq. (2.2) agrees with the Mises plasticity condition.

As was noted above, pore growth is determined by local phenomena and the effective stress $\langle\sigma_{ij}\rangle$ should be calculated by averaging the respective fields over the ensemble of their realizations in the neighborhood of the pores. Following [8], this latter can be represented in the form of a sum of active (macroscopic) and microscopic components. Since the exact estimate of the microstress fields is not possible, we then introduce a mean-statistical value of the microstresses $\langle\sigma'_{ij}\rangle$, for the macroscopic approach taken, which characterizes a definite property of the material under consideration similarly to the yield point σ_s. Taking account of the above, the expression for the effective stresses in the neighborhood of the pores in w is written in the form

$$\langle\sigma_{ij}\rangle = \sigma_{ij} + \langle\sigma'_{ij}\rangle. \tag{2.3}$$

Analogously for the strain rates governing the change in pore size and shape in w, we will have

$$\langle\xi_{ij}\rangle = \xi_{ij} + \langle\xi'_{ij}\rangle = \eta\xi_{ij} \quad (\eta = 1 + \langle\xi'_{ij}\rangle / \xi_{ij}). \tag{2.4}$$

The σ_{ij}, ξ_{ij} in (2.3) and (2.4) are macroscopic, but the $\langle\sigma'_{ij}\rangle$, $\langle\xi'_{ij}\rangle$ are averaged microscopic stresses and strains, respectively; η is a material parameter characterizing the inhomogeneity of the strain state within the elementary structural cells Δw. The case $\langle\sigma'_{ij}\rangle = 0$ evidently corresponds to the Reiss approximation, and $\eta = 1$ to the Voight approximation [10].

Taking (2.2) as the plastic potential and using (2.3) and (2.4), we determine the strain rate in w

$$\xi_{ij} = \frac{\lambda}{\eta}\frac{\partial f}{\partial\langle\sigma_{ij}\rangle} = \frac{\lambda}{\eta}\left[\gamma\left(\sigma_{ij} + \langle\sigma'_{ij}\rangle\right) + 6\delta_{ij}\alpha\left(\sigma + \langle\sigma'\rangle\right)\right]\ (\lambda \geqslant 0), \tag{2.5}$$

where σ, $\langle\sigma'\rangle$ are the hydrostatic pressures of the macroscopic and microscopic stress component (2.3), respectively, δ_{ij} is the Kronecker delta. Evaluating the shear strain rate intensity from (2.5)

$$H = (2\xi_{ij}\xi_{ij})^{1/2} = 2\lambda\gamma T/\eta,$$

we have

$$\lambda = H\eta/2\gamma T.$$

The equations for the strain rate are then written in the form

$$\xi_{ij} = \frac{H\eta}{2T}\left(\sigma_{ij} + \langle\sigma'_{ij}\rangle - \sigma - \langle\sigma'\rangle\right) + \delta_{ij}\frac{3\alpha H\eta}{\gamma T}\left(\sigma + \langle\sigma'\rangle\right),$$

and for the rate of change of volume caused by pore growth

$$\xi = \delta_{ij}\xi_{ij} = 9\alpha(\sigma + \langle\sigma'\rangle)H\eta/\gamma T. \tag{2.6}$$

Using the relationship [9]

$$\xi = \frac{d\varepsilon}{dt} = \frac{1}{1-v}\frac{dv}{dt},$$

we find the kinetic equation of the change in material porosity during its plastic deformation from (2.6)

$$dv/dt = 9(1 - v)\alpha H\eta(\sigma + \langle\sigma'\rangle)/\gamma T. \tag{2.7}$$

Since v << 1 for viscous fracture, and the porosity function is [9]

$$\alpha(v) = c(\ln v)^{-2},$$

then to the accuracy of second-order terms, the kinetic equation (2.7) becomes

$$(\ln v)^2\, dv = 3\sqrt{3}\, c\eta \frac{\sigma + \langle\sigma'\rangle}{\sigma_s}\, d\Gamma, \tag{2.8}$$

wehre $d\Gamma = Hdt$ is the increment in shear strain intensity, and c is the pore shape factor that can vary during deformation between the values $c = 1/4$ for spherical pores and $c = 1/3$ for cylindrical pores.

Integrating (2.8) finally yields

$$\int_0^{\Gamma} \eta c \frac{\sigma + \langle\sigma'\rangle}{\sigma_s}\, d\Gamma = F(v) - F(v_0) = \Delta F, \tag{2.9}$$

$$\text{where } F(v) = \frac{v}{3\sqrt{3}}\left[(\ln v - 1)^2 + 1\right].$$

In conformity with the above, the limit state (macroscopic fracture in w) will be determined by the condition for reaching the critical porosity v*:

$$\int_0^{\Gamma^*} \eta c \frac{\sigma + \langle\sigma'\rangle}{\sigma_s}\, d\Gamma = \frac{1}{3\sqrt{3}}\{v^*[(\ln v^* - 1)^2 + 1] - \tag{2.10}$$
$$- v_0[(\ln v_0 - 1)^2 + 1]\} = \Delta F^*.$$

Equation (2.10) for the limiting shear strain intensity Γ^* corresponds to the viscous fracture criterion for plastic molding of metals. The quantities v_0, v^* therein are the initial and limit degrees of material damage by the pores (the relatively slight influence of the pore shape can be noted since $0.25 \leq c \leq 0.33$), η, $\langle\sigma'\rangle$, σ_s characterize definite macroscopic properties of the material which depend on its structure, history, and strain conditions in the general case; and σ, Γ are the parameters of the strain process. The absence of macrofracture corresponds to the condition

$$\int_0^{\Gamma} c\eta \frac{\sigma + \langle\sigma'\rangle}{\sigma_s}\, d\Gamma = \Delta F \leqslant \Delta F^*. \tag{2.11}$$

Compliance with (2.11) should be verified by integrating its left side along the loading path of fixed points of the volume V. Different points of the plastic deformation focus at definite times are thereby characterized by a damage parameter ΔF, whose critical values domains ΔF^* correspond to the macrocrack location and shape.

3. To integrate (2.10) and (2.11), it is necessary to give the functions

$$\langle\sigma'\rangle = \varphi\left(\int d\Gamma\right),\ \eta = f\left(\int d\Gamma\right),\ \sigma_s = \psi\left(\int d\Gamma\right), \tag{3.1}$$

which are assumed monotonically increasing and constrained. For a macroscopic description of the fracture process and because of the complexity of the theoretical buildup of relations (3.1), these latter should be determined experimentally. At the present time, the dependence of σ_s on different factors has been studied in detail, however, there are only individual estimates of the inhomogeneous microstress and microstrain fields in real metals. To clarify certain qualitative singularities in their distribution, let us consider a simple model of an individual grain containing a pore 1, rigid inclusion 2, and blocks a, b, c (Fig. 1). As noted above, pore formation and growth occur on the grain and the mentioned domain boundaries (covered in Fig. 1 by a double hatching). Experimental studies performed by the method of x-ray diffraction [11] showed that the appropriate domains are among the "strong" (in contrast to the "weak") matrix domains and the microstresses originating therein can significantly exceed the mean values of the micro- and macrostresses within the grain. The experimentally established fact of an oriented macrostress distribution under plastic deformation, according to which the hydrostatic microstress field component is tensile in "strong" and compressive in "weak" domains independently of the sign of the applied macrostresses, is important (an analogous theoretical result is obtained in [12]). It hence follows that the component $\langle\sigma'\rangle$ should also be tensile. On the other hand, the presence of "strong" domains retarding shear propagation corresponds to $\eta < 1$, in conformity with (2.4), where the value of η should increase as plastic flow develops when the microelastic strains become small compared to the macroscopic strains. Since the influence of the strain history on the instantaneous macroscopic properties is here related directly to the microstresses, the nature of the change in the functions φ and ψ in (3.1) should be analogous.

The estimates presented permit taking

$$\eta c \simeq \text{const}, \quad \frac{\langle\sigma'\rangle}{\sigma_s} = \frac{\varphi\left(\int d\Gamma\right)}{\psi\left(\int d\Gamma\right)} \simeq \text{const} = A,$$

as the first approximation in (2.10) and (2.11), and the viscous fracture criterion is written in the simplest form

$$\int_0^{\Gamma^*}\left(\frac{\sigma}{\sigma_s} + A\right)d\Gamma = \frac{\Delta F^*}{c\eta} = \Delta F_1^*, \tag{3.2}$$

which agrees with empirical ultimate plasticity conditions established experimentally in [13, 14] for metals and porous bodies.

As follows from (2.7), the pore growth $dv/dt > 0$ holds for $\sigma + \langle\sigma'\rangle > 0$, while $dv/dt < 0$ for $\sigma + \langle\sigma'\rangle < 0$, and the material porosity diminishes. Hence, the value of the ultimate shear strain intensity Γ^*, corresponding to the solution of (2.10) and (3.2), is a function of their integrands. To analyze the situations occurring here, we consider the simple loading case when the effects of a change in the strain and unloading directions do not hold, and therefore σ/σ_s = const. Integrating (3.2) then yields

$$\Gamma^* = \frac{\Delta F_1^* \sigma_s}{\sigma + \langle\sigma'\rangle}. \tag{3.3}$$

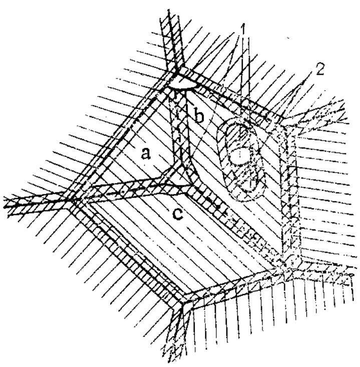

Fig. 1

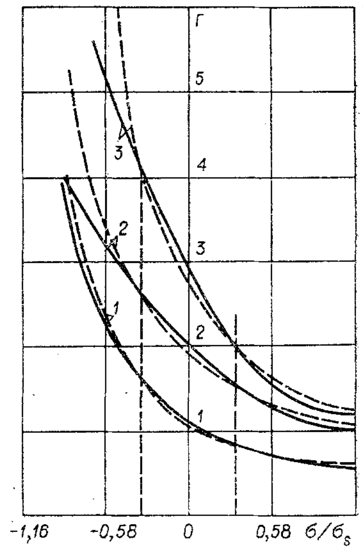

Fig. 2

The solution of (3.3) has meaning for $\sigma > -\langle\sigma'\rangle$, when the material porosity grows and the critical value v^* can be achieved. As the tensile hydrostatic pressure diminishes, the limit shear strain intensity Γ^* increases and tends to the maximum value ($\Gamma^* \to \infty$) for compressive hydrostatic pressures $\sigma < -\langle\sigma'\rangle$. The plastic strain is here accompanied by welding of the pores and the fracture mechanism under consideration does not hold. The mentioned phenomena are well-known experimentally.

To confirm criterion (3.3) and to estimate certain macroscopic material characteristics, experimental plasticity diagrams presented in [3] for different metals and alloys were used. The parameters ΔF_i^* and $\langle\sigma'\rangle/\sigma_s$ were calculated from (3.3) for experimental values of Γ^*, corresponding to tension and compression tests on cylindrical specimens ($\sigma/\sigma_s = \pm\ 0.33$). The results for St. 45, the titanium alloy VT 1-1, and the aluminum alloy AMg 2 are presented in Fig. 2 (curves 1-3, respectively, the solid curves refer to experimental results [3] while the dashed curves are constructed from (3.3)). The good agreement with experiment can be seen even for the simplified versions of the criterion (2.10). The calculated values of the material parameters were $\langle\sigma'\rangle/\sigma_s = 1.08$, $\Delta F_i^* = 1.2$ for steel 45, $\langle\sigma'\rangle/\sigma_s = 1.3$, $\Delta F_i^* = 2.5$ for the alloy VT 1-1, and $\langle\sigma'\rangle/\sigma_s = 0.93$, $\Delta F_i^* = 2.52$ for the alloy AMg 2. However, it must be noted that the parameters c, η, ΔF_i^*, and v_0, v^* cannot, in conformity with (3.2) and (2.10), be calculated independently of the solution of an equation of type (3.3) and experimental results of a different nature should be used to determine them.

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